

Chapter 18  
Magnetic Fields Chapter Review

EQUATIONS:

- $\mathbf{F}_{on\ pt.\ chg.} = q\mathbf{v}\times\mathbf{B}$  [This is the force a point charge  $q$  experiences when moving with velocity  $\mathbf{v}$  in a magnetic field  $\mathbf{B}$ . The expression is usually used to determine the magnitude of the force. The direction is determined using the right hand rule (after all, you are dealing with a cross product). Note that the direction of the magnetic force (i.e., the direction of the cross product) is perpendicular to the plane defined by the velocity and magnetic field vectors.]
- $\mathbf{F}_{on\ current\ carrying\ wire} = i\mathbf{L}\times\mathbf{B}$  [This is the force a current-carrying wire feels due to its presence in a magnetic field, where  $i$  is the current in the wire,  $\mathbf{L}$  is a vector equal in magnitude to the length of the wire and oriented in the direction of the current, and  $\mathbf{B}$  is the magnetic field. Again, this expression is usually used to determine the magnitude of the force. The direction is determined using the right hand rule. Note that this relationship assumes that the magnetic field is constant over the entire wire.]
- $\mathbf{F}_{tot} = q\mathbf{v}\times\mathbf{B} + q\mathbf{E}$  [This is Lorentz's expression. It identifies all of the forces that could possibly act on a charge  $q$  including the force associated with a magnetic field  $\mathbf{B}$  and the force associated with an electric field  $\mathbf{E}$ . Note that just as  $\mathbf{E}$  and  $\mathbf{B}$  do not necessarily have to be in the same direction, neither do the forces produced by  $\mathbf{E}$  and  $\mathbf{B}$  have to be in the same direction.]
- $B = \frac{\mu_o i}{2\pi r}$  [This is the expression for the magnitude of the magnetic field produced by a current-carrying wire. The constant  $\mu_o$  is called the permeability of free space and is equal to  $4\pi \times 10^{-7}$  tesla-meters/amp. The DIRECTION of the wire's magnetic field will be tangent to a circle centered on the wire. To determine whether the direction is counterclockwise or clockwise, use the right thumb rule (i.e., with the right thumb directed along the line of the current, the curl of the fingers yields the clockwise/counterclockwise sense of the magnetic field).]
- $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o i_{thru\ the\ path's\ face}$  [This is Ampere's Law. It states the following: Draw an appropriately defined, imaginary Amperian path (as was the case with Gauss's Law, symmetry matters). Identify a differential length  $d\mathbf{l}$  on the path and dot  $d\mathbf{l}$  into the magnetic field vector  $\mathbf{B}$  evaluated at  $d\mathbf{l}$ . Sum that quantity via an integral around the Amperian path (this sum is called the circulation of  $\mathbf{B}$ ). According to Ampere, that quantity will be proportional to the amount of current that passes through the face of the path. The proportionality constant needed to make the relationship into an equality is  $\mu_o$ .]

- $$d\mathbf{B} = \frac{\mu_o i}{4\pi} \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$
 [This is the Law of Biot Savart. It is used to derive the magnetic field produced at a point of interest due to the presence of a current-carrying wire. It works as follows: Choose an arbitrary section of wire and define a differential length  $dl$ , where the current direction and the direction of  $dl$  are the same. Define a position vector  $r$  that goes from  $dl$  to the point of interest. The execution of Biot Savart produces an expression for the differential magnetic field  $d\mathbf{B}$  at the point of interest. With appropriate integration, the net magnetic field at that point may be determined.]
- $$dB = \frac{\mu_o i}{4\pi} \frac{dl \sin\theta}{r^2}$$
 [This is the Law of Biot Savart after the cross product has been executed. It gives you the magnitude of the differential magnetic field  $d\mathbf{B}$  produced by the current  $i$  in a section of wire of differential length  $dl$ . The direction of the differential magnetic field is determined using the right thumb rule. If the direction of that differential field is the same as the direction of all other differential fields due to all of the other segments, all you have to do is integrate to determine the net field. If the differential fields due to various segments are in different directions, you will have to break  $d\mathbf{B}$  into its component parts and integrate each component. Note that if it is obvious, due to symmetry, that a particular component will add to zero with the integration, ignore that component.]
- $$B_{coil} = \mu_o ni_o$$
 [This is the magnitude of the magnetic field down the axis of a current-carrying coil whose turn ratio (i.e., whose number of winds per unit length) is  $n$ .]
- $$\mu_m$$
 [This is the magnetic dipole moment vector associated with a current-carrying coil. Its magnitude is defined as  $iNA$ , where  $i$  is the current in the coil,  $N$  is the number of winds in the coil, and  $A$  is the area of the coil's face.]
- $$\Gamma = \mu_m \times \mathbf{B}$$
 [A current-carrying coil pinned in a magnetic field will be motivated to rotate by a torque generated by the interaction of the current and the magnetic field. The torque will equal the cross product of the magnetic dipole moment vector  $\mu_m$  and  $\mathbf{B}$ . THIS IS NOT A RELATIONSHIP YOU WILL USE MUCH.]
- $$U = -\mu_m \cdot \mathbf{B}$$
 [A current-carrying coil pinned in a magnetic field will be motivated to rotate due to the interaction of the current and the magnetic field. As the work producing torque is conservative, a potential energy function can be associated with the coil in the field. This function will be equal to the opposite of the dot product of the magnetic dipole moment vector  $\mu_m$  and  $\mathbf{B}$ . THIS IS NOT A RELATIONSHIP YOU WILL USE MUCH.]

COMMENTS, HINTS, and THINGS to be aware of:

- If you put an initially stationary, charged particle in an electric field, its velocity magnitude will change as the particle accelerates due to the presence of the field. If you put an initially stationary, charged particle in a magnetic field, its velocity magnitude will NOT change due to the presence of the field. Electric fields are modified force fields (remember, they are defined as  $F/q$ ). Magnetic fields are NOT modified force fields.

- If you put a moving charged particle in an electric field, its velocity magnitude will change depending upon the direction of the field. If you put a moving charged particle in a magnetic field, its velocity magnitude will not change but its DIRECTION MAY CHANGE due to the presence of the field. That is, magnetic forces are centripetal forces. They don't change a velocity's magnitude, but they will change a velocity's direction.
- What makes magnetic fields so difficult to deal with is the fact that they aren't linear in nature. The direction of an electric field produced by, say, a positive charge, is radially outward from the charge. The direction of a magnetic field produced by, say, a wire carrying a current oriented out of the page is TANGENT TO A CIRCLE centered on the wire and directed counterclockwise (remember, the sense of the B-field's direction--whether clockwise or counterclockwise--is determined using the right thumb rule).
- Gauss's Law says that the electric flux through a closed surface (i.e.,  $\int_S \mathbf{E} \cdot d\mathbf{S}$ ) will be proportional to the charge enclosed within the surface. Ampere's Law says that the circulation of the magnetic field around a closed path (i.e.,  $\oint \mathbf{B} \cdot d\mathbf{S}$ ) is proportional to the amount of current that flows through the face of the path (i.e., through the area defined by the path). Just as Gauss's Law is useful when symmetry can be exploited, so also is the case with Ampere's Law. The difference is that the symmetry needed for Gauss's Law is associated with spherical or cylindrical surfaces, whereas the symmetry needed for Ampere's Law is associated with circular or rectangular paths.
- Let's assume you want to determine the magnetic field set up by some configuration of current-carrying wires. When can you use Ampere's Law to do so?
  - If you can define a closed path upon which the magnitude of the magnetic field is constant, Ampere's Law will work.
  - If you can define a closed path upon which the magnitude of the magnetic field is constant on part of the path and  $\mathbf{B} \cdot d\mathbf{l}$  is zero for the rest of the path, Ampere's Law will work.
- Let's assume you want to determine the magnetic field set up by some configuration of current-carrying wires. When can you use the Law of Biot Savart to do so?
  - If you are dealing with current-carrying wires in a situation in which there is no symmetry, or in a situation in which the symmetry is odd (a wire configuration that has the outline of a basket, for instance), Biot Savart will work.
  - If you are dealing with a situation in which you cannot define a closed path upon which the magnitude of the magnetic field is constant, Biot Savart will still work.
- You may run into situations in which you have to determine the magnetic field due to current spread out through an extended object. To do so, determine the magnitude and direction of a differential bit of current flow, break that field into component parts, then integrate to determine the whole field due to the whole current.
- A galvanometer is an ammeter that swings full deflection when  $5 \times 10^{-4}$  amps (i.e., a half milliamp) flows through it.
- An ammeter designed to deal with currents larger than  $5 \times 10^{-4}$  amps is nothing more than a galvanometer in parallel with a shunt resistor of appropriate size.

- A voltmeter is nothing more than a galvanometer in series with an extra resistor of appropriate size.